Reg. No.:

Name :

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Mid-Term Examinations, December 2020

Programme	:	**B.Tech**	Semester	:	**Fall 2020-2021**
Course	:	**Calculus and Laplace Transforms**	Code	:	**MAT1001**
Faculty	:	**Dr. Suresh Dara**	Slot/Class No.	:	**B21+B22+B23/1501**
Time	:	**1½ hours**	Max. Marks	:	**50**

Answer all the Questions

Q. No.	Question Description	Marks
1	Discuss the continuity of the function at the origin $f(x,y) = \begin{cases} \frac{2x(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$	**10**
ANS	Step-1: well-defined --- 2 M Step-2: Existence of Limit -- 4 M Step-3: Limit=f(a,b) -- 2 M Conclusion --- 2 M Continuous at origin	
2	A manufacturer's production modeled by Cobb-Douglas function $f(x,y) = 100\, x^{\frac{3}{4}}\, y^{\frac{1}{4}}$ where x represents the units of labor and y represents the units of capital. Each labor unit cost costs 150 rupees and each capital unit costs 250 rupees. The total expenses for labor and capital cannot exceed 50,000 rupees. Will the maximum production level exceed 16,000 units?	**10**
	$\phi = 150x + 250y - 50000$ $F = f + \lambda\phi$ $\frac{\partial F}{\partial x}$=0 ---eq1 $\frac{\partial F}{\partial y}$=0 --- eq2 $\phi = 0$ ----eq3 --- 2M From eq1---- get λ -- 2M From eq2 ---- get the value of x in y--------------------------------------- 2M From eq3---- get the value of y= 50 and then the value of x=250 ----- 2M Then f(250, 50)>16,000 Conclusion -- 2M	

3	Reverse the order of integration, and evaluate the integral. $$\int_0^{\frac{1}{16}}\int_{y^{\frac{1}{4}}}^{\frac{1}{2}} \cos(16\pi x^5)\,dx\,dy$$	**10**
	Reversing the order of integration with proper limits ------------------------------------- 4 M $$\int_{x=0}^{\frac{1}{2}}\int_{y=0}^{x^4}\cos(16\pi x^5)\;dy\,dx$$ For evaluation of integration -- 6 M $$\frac{1}{80\pi}$$	
4	Find the volume of the region enclosed by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $y+z=4$.	**10**
	For Limits -- 4 M $$\int_{-2}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{z=0}^{4-y} dz\,dy\,dx \quad or \quad \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\int_{z=0}^{4-r\sin\theta} r\,dz\,dr\,d\theta$$ For each integration 2M each-- 6 M $$16\,\pi$$	
5	If $F=(2x^2-3z)i-2xyj-4xk$, evaluate a) $\int_V \nabla.F\,dV$ b) $\int_V \nabla\times F\,dV$ Where V is the closed region bounded by $x=0, y=0, z=0,$ $2x+2y+z=4.$	**10**
	For limits -- 2 M $$\int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y}$$ $\nabla.F$ = 2x--- 2 M $\nabla\times F=j-2yk$ -- 2 M First integration = 8/3--- 2 M Second Integration = $\frac{8}{3}(j-k)$--- 2 M	

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